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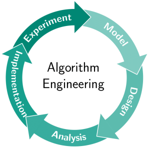

Trusted Scalable SAT Solving with on-the-fly LRAT Checking

SAT 2024, Pune, India

Dominik Schreiber | August 22, 2024

Motivation

Distributed clause-sharing solvers push the frontier of feasible problems.

- Many sequential CDCL solvers run in parallel
- Careful exchange of useful conflict clauses
- Mean speedup of 419 @ 3072 cores for difficult instances [SS24]

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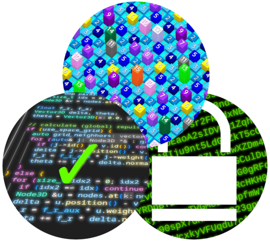

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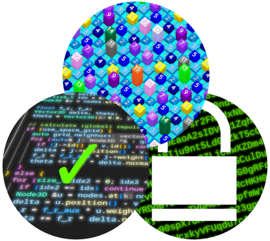
Parallel & distributed solvers are harder to trust than sequential solvers.

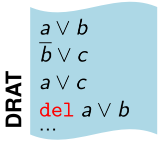
- Large technology stack leaves more room for bugs, errors
- More difficult and expensive to test rigorously
- Fragile – a single bit flip in a clause can induce a wrong result



The Story Thus Far

Producing proofs from parallel clause sharing is challenging.

- Popular DRAT format does not scale in parallel settings [HMP14; FB22]



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DRAT

$a \lor b$
$\overline{b} \lor c$
$a \lor c$
del $a \lor b$
…

LRAT

$1765 : a \lor b \mid 823, 1277$
$1766 : \overline{b} \lor c \mid 1338, 54$
$1767 : a \lor c \mid 1765, 1766$
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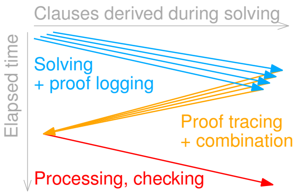

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  1. Write individual partial proofs during solving
  2. Rewind solving + sharing, funnel required derivations into single file
  3. Check combined proof file
- **Bottleneck:** sequential assembly and checking of monolithic proof
  - Throttled by I/O bandwidth at final process
  - Sometimes hundreds of Gigabytes of proof information
  - Proof production + checking @ 1520 cores takes $\approx 3\times$ solving time (latest setup – submitted to JAR)
  - Intuition "*If solving fits into RAM, checking will as well*" no longer holds

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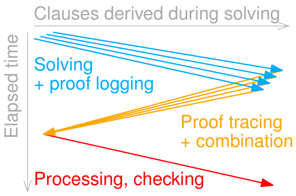

On-the-fly Checking with Sequential Solvers

Marijn Heule: Since LRAT checking is so efficient, we can feasibly do it in realtime!

```
mkfifo lratproof.pipe  // create “pipe” file

// Solve & check concurrently via pipe
./solver input.cnf lratproof.pipe &
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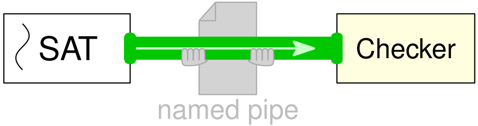

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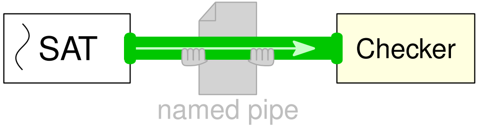

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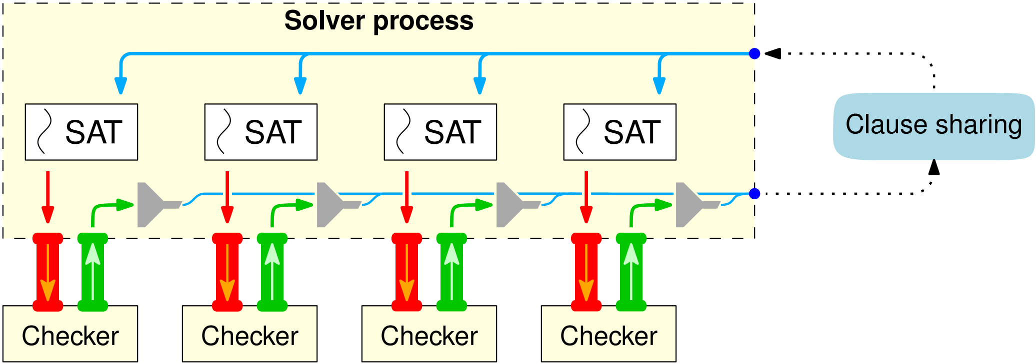
- Program code indistinguishable from plain file I/O (only difference: `mkfifo`)
- Does not yield a persistent artifact to validate by independent parties

A First Parallel & Distributed Setup



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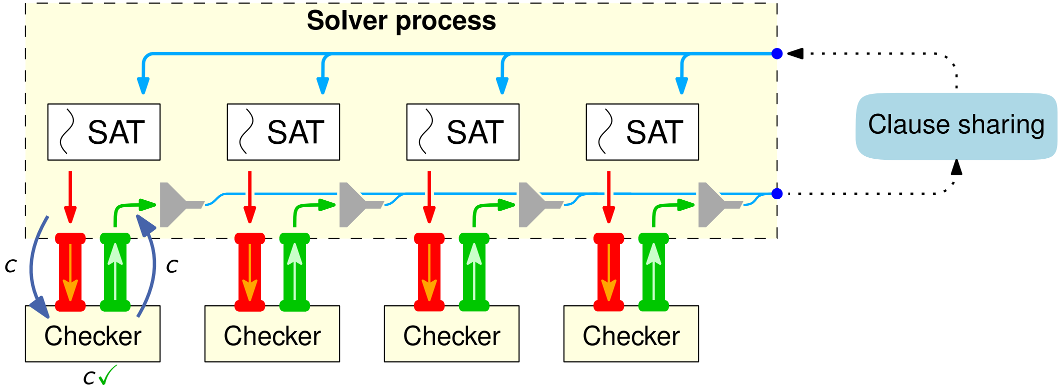

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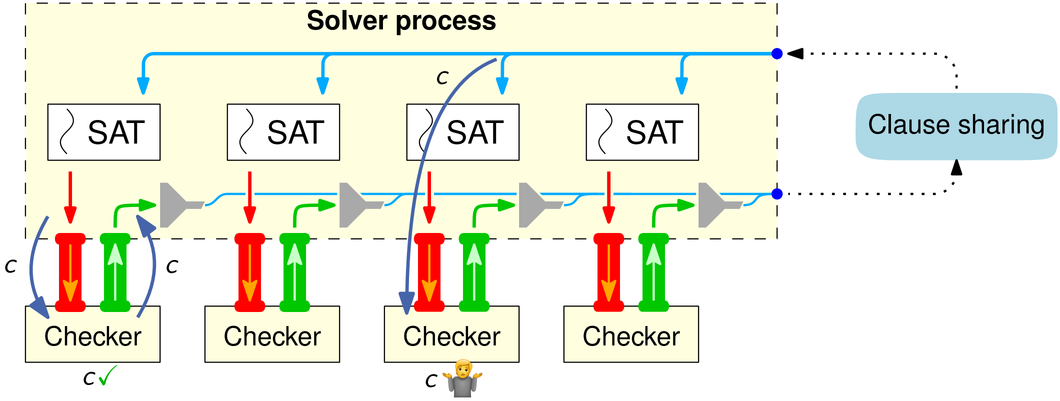

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Which components do we still need to trust?

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Goal: Only need to trust the parser and checkers, nothing else!

Signatures (1/2)

Clause $\xrightarrow{\mathcal{S}}$ Signature

ID: 159514 | Lits: $4 \lor \overline{163} \lor \overline{145} \lor \overline{28} \lor 158$ → `d12e6a68fc3456e95d64a735555783d6`

Assumption: Parser and checkers know a "secret" signature function $\mathcal{S}$

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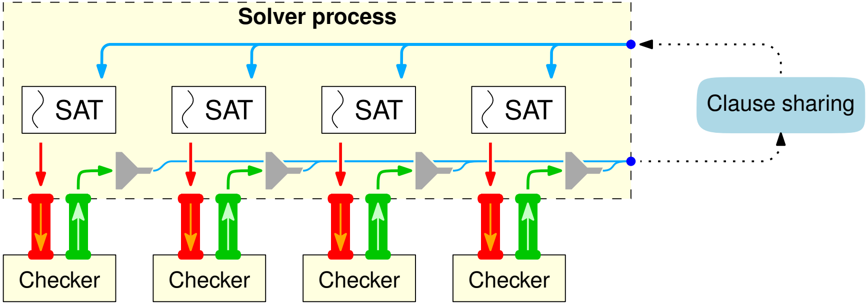

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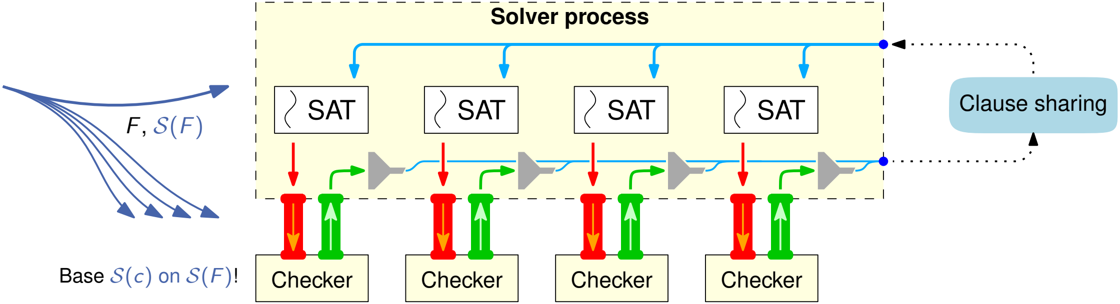

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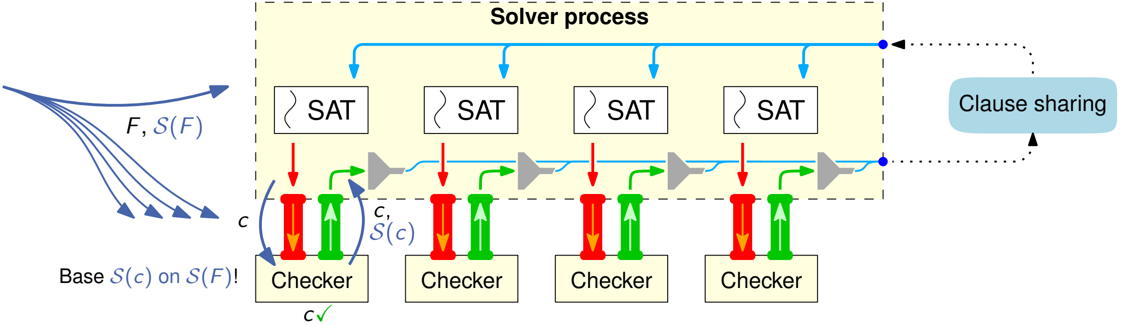

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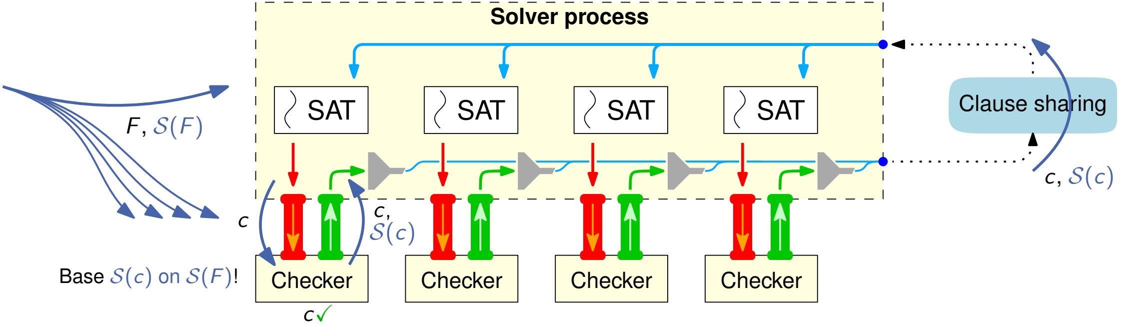

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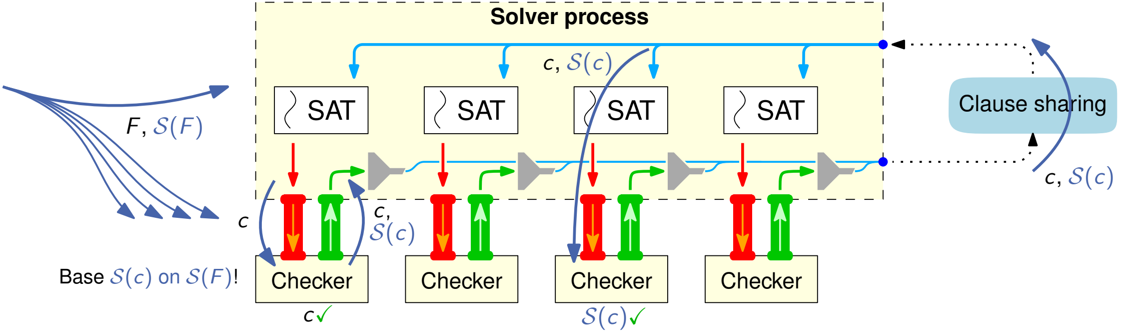

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$$\mathcal{S}(F) := H_K(F \mid\mid 0_{(2\text{ bytes})}) , \quad \mathcal{S}(c) := H_K\big(id(c) \mid\mid c \mid\mid \mathcal{S}(F)\big) , \quad \mathcal{S}(\bot) := H_K\big(20_{(1\text{ byte})} \mid\mid \mathcal{S}(F)\big)$$

Confidence

What breaks our approach?

Obtain $\mathcal{S}(\bot)$ for satisfiable F

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Find $(c, \mathcal{S}(c))$ where c is unsound w.r.t. F → Obtain $\mathcal{S}(\perp)$ for satisfiable F

→ : "enables"

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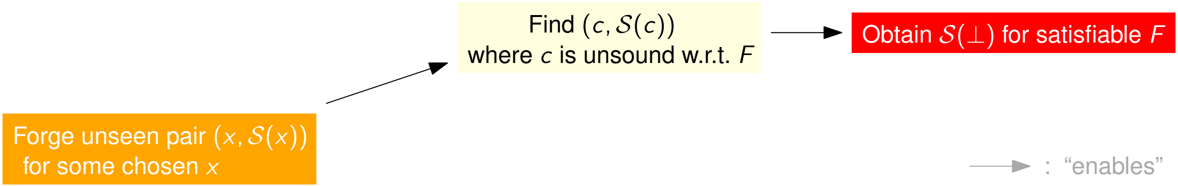

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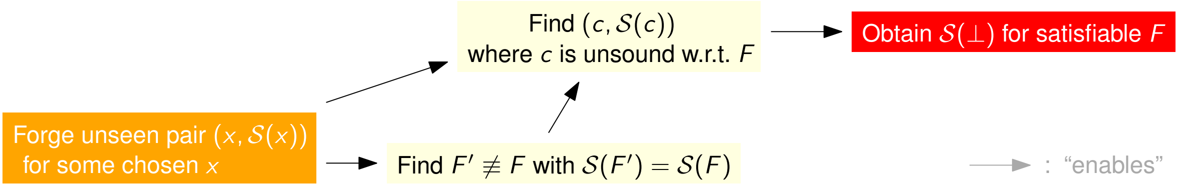

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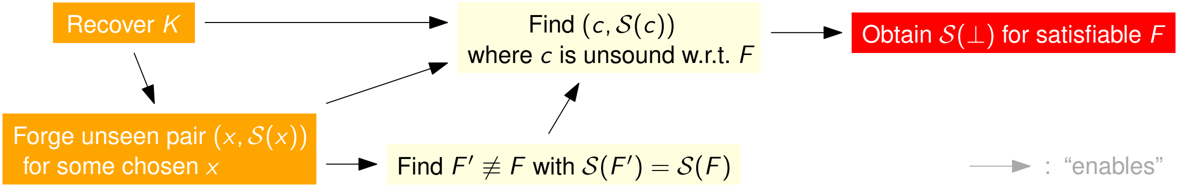

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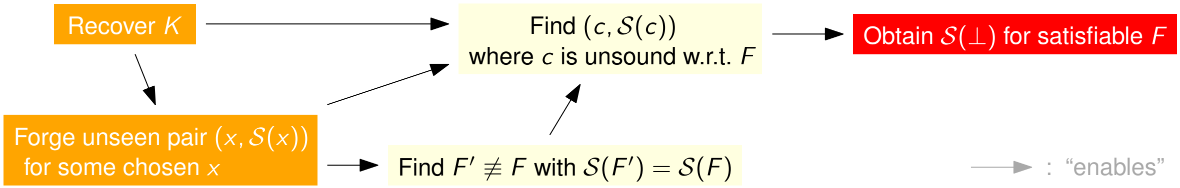

Security Claims of 128-bit SipHash

Forging a previously unseen pair $(x, \mathcal{S}_K(x))$ succeeds with probability $2^{-128} \approx 10^{-38}$.
Recovering K succeeds with probability 2^{-128}.

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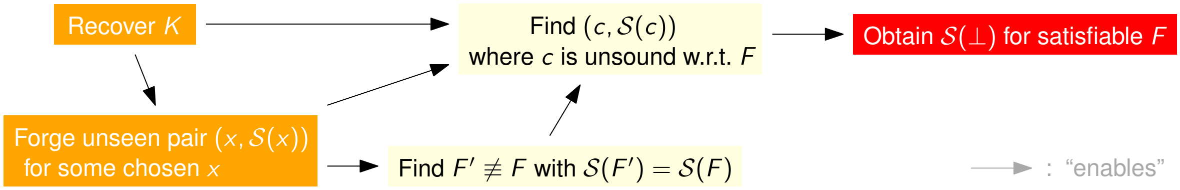

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Intuition: Inadvertent bugs / errors / faults during solving "can't do better" than deliberate attacks!

Implementation

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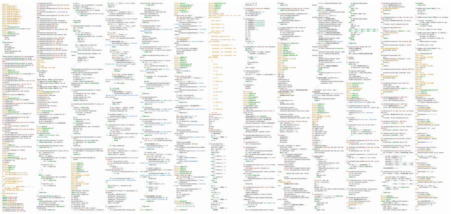


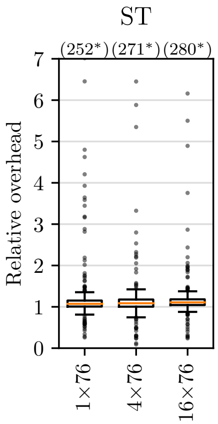
- Distributed framework: MALLOBSAT [SS24]
- Sequential solver: CADICAL with LRAT output [PFB23]
- Trusted modules: Parser, checker, confirmer
  - Confirmer takes F and $\mathcal{S}(\bot)$, validates $\mathcal{S}(\bot)$
  - Overall $\approx$ 1k effective lines of C99 code

Setup

- $\leq$ 32 compute nodes of HPC cluster HoreKa
  - Per node: 2×38 cores (76 hardware threads), 256 GB RAM
- SAT Competition 2023 benchmarks
- Time limits: 300 s wallclock time for solving,
  1500 s for postprocessing + checking

Overhead Relative to Proof-free Solving

Monolithic proofs [Mic+23]



Overhead relative to solving time w/o LRAT outputs · ST: Solving time · TuP: Time until Proof present · TuV: Time until Validation done
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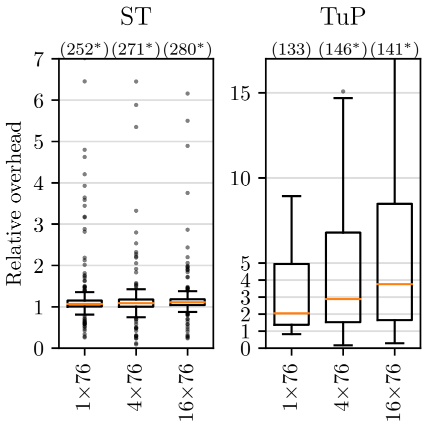

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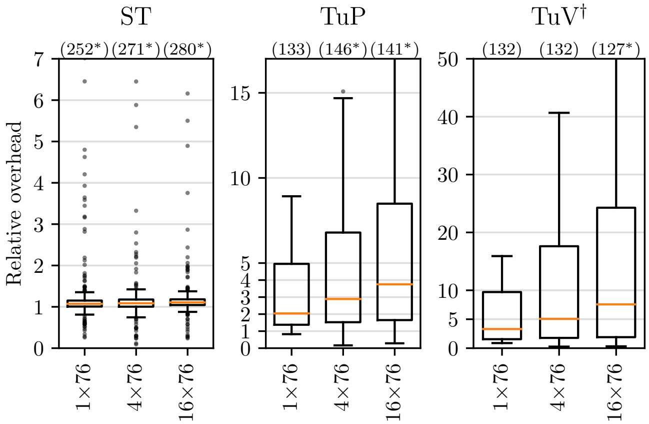

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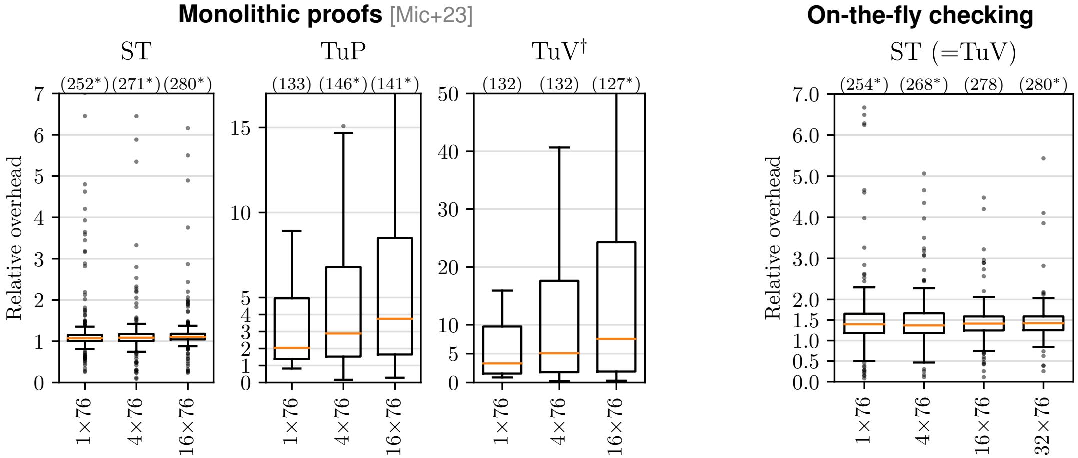

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- ✓ Generic framework
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  - Independent of structure, implementation of clause exchange
- ✓ Extended to checking satisfying assignments
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! High memory usage (+60% compared to proof-less solving)
  - Compressing clauses in checkers?
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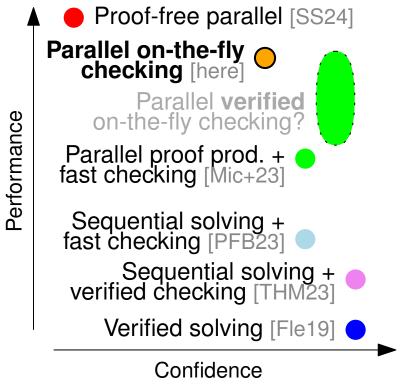
? Formal verification of trusted processes?

Cooperation wanted!

  - Would result in first verified distributed SAT solver (in terms of correctness, not termination)
  - Extend projects like `cake_lpr` [THM23]? Efficient enough?
  - Verify (parts of) C99 codebase? BMC? Verified compilation?

Conclusion

- Bottleneck-free approach to on-the-fly proof checking for distributed clause-sharing solving
- Trusted parties: Isolated parser and checker processes, extending usual LRAT checking interface
- Saves an order of magnitude in running time overhead over explicit proof production
- Paves the road to verified distributed SAT solving



github.com/domschrei/impcheck

References

[AB12] Jean-Philippe Aumasson and Daniel J. Bernstein. "SipHash: a fast short-input PRF". In: *International Conference on Cryptology in India*. Springer. 2012, pp. 489–508. DOI: 10.1007/978-3-642-34931-7_28.

[FB22] Mathias Fleury and Armin Biere. "Scalable Proof Producing Multi-Threaded SAT Solving with Gimsatul through Sharing instead of Copying Clauses". In: *Pragmatics of SAT*. 2022.

[Fle19] Mathias Fleury. "Optimizing a verified SAT solver". In: *NASA Formal Methods: 11th International Symposium, NFM 2019, Houston, TX, USA, May 7–9, 2019, Proceedings 11*. Springer. 2019, pp. 148–165.

[HMP14] Marijn J. H. Heule, Norbert Manthey, and Tobias Philipp. "Validating Unsatisfiability Results of Clause Sharing Parallel SAT Solvers.". In: *Pragmatics of SAT*. 2014, pp. 12–25. DOI: 10.29007/6vwg.

[Mic+23] Dawn Michaelson et al. "Unsatisfiability proofs for distributed clause-sharing SAT solvers". In: *Tools and Algorithms for the Construction and Analysis of Systems (TACAS)*. Springer. 2023, pp. 348–366. DOI: 10.1007/978-3-031-30823-9_18.

[PFB23] Florian Pollitt, Mathias Fleury, and Armin Biere. "Faster LRAT checking than solving with CaDiCaL". In: *Theory and Applications of Satisfiability Testing (SAT)*. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2023. DOI: 10.4230/LIPIcs.SAT.2023.21.

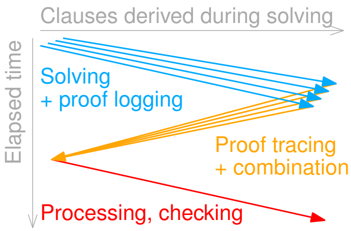
[SS24] Dominik Schreiber and Peter Sanders. "MallobSat: Scalable SAT Solving by Clause Sharing". In: *Journal of Artificial Intelligence Research (JAIR)* (2024). In press.

[THM23] Yong Kiam Tan, Marijn J. H. Heule, and Magnus Myreen. "Verified LRAT and LPR Proof Checking with cake_lpr". In: *SAT Competition*. 2023, p. 89. URL: https://researchportal.helsinki.fi/files/269128852/sc2023_proceedings.pdf.

Intrinsic Scalability Issues

Bottleneck: sequential assembly and checking of monolithic proof

- Throttled by I/O bandwidth at final process
- Sometimes hundreds of Gigabytes of proof information
- Proof production + checking @ 1520 cores takes $\approx 3\times$ solving time
  (latest setup – submitted to JAR)
- Intuition "*If solving fits into RAM, checking will as well*" no longer holds



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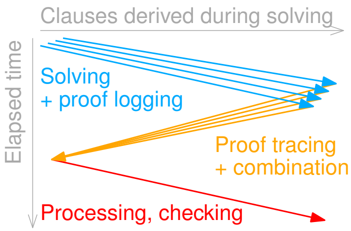
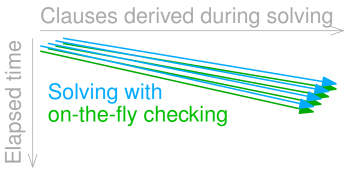


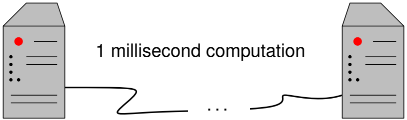
Our aim: Make checking scalable by dropping requirement of a single, persistent proof



The (Un)Likelihood of 2^{-128}

- Estimated (2007) probability of dying due to a **local** comet/asteroid impact: 1 in 5 700 000[1]

  [1] http://www.boulder.swri.edu/clark/binhaz07.ppt
- Average human life span estimate (conservative): 80 years
- Probability of such an impact per millisecond: 1 in $5\,700\,000 \cdot (80 \cdot 365 \cdot 24 \cdot 3600 \cdot 1000) \approx 1.4 \cdot 10^{-19}$
- Two unrelated impacts in the same millisecond: $10^{-19} \cdot 10^{-19} = 10^{-38} \approx 2^{-128}$



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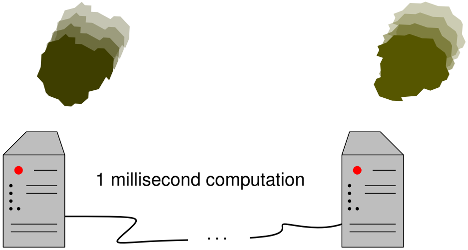

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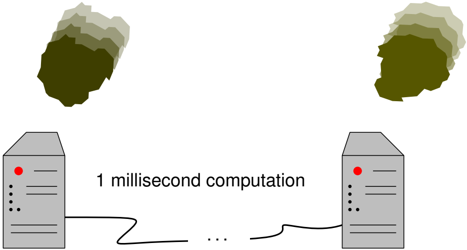
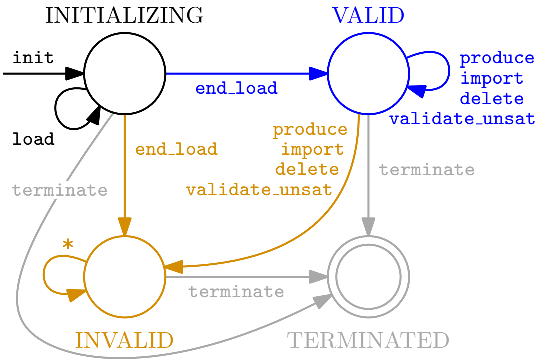


- Same argument with cosmic radiation flipping two particular bytes (prob. 10^{-15} per byte per sec.), causing a formally verified checker to hallucinate unsatisfiability

Checker Interface

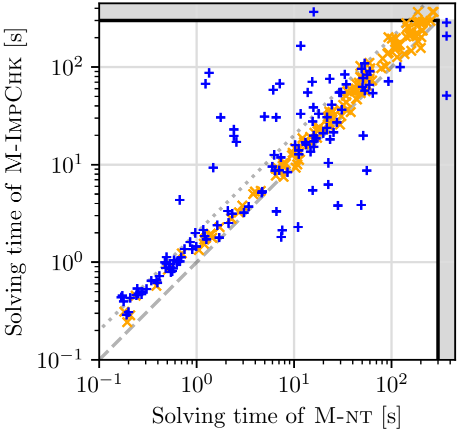
Protocol realized via named pipes:

```
init(sig: Signature) → void
load(formula: ClauseSet) → void
end_load() → bool
produce(id: ID, lits: Clause, hints: IDList, share: bool)
      → (bool, Signature?)
import(id: ID, lits: Clause, sig: Signature) → bool
delete(ids: IDList) → bool
validate_unsat() → (bool, Signature?)
terminate() → void
```

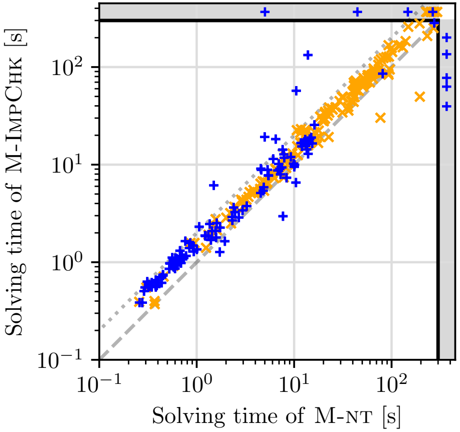


Results: Solving Time Overhead

1 node (76 cores)



32 nodes (2432 cores)



M-nt: MallobSat+CaDiCaL, no LRAT output · M-ImpChk: MallobSat+CaDiCaL + on-the-fly checking

Results: Solving Times (w/o Assembly, Checking)